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ABSTRACT

An option-pricing model is developed to rank investments that might improve water quality. The model presumes that two investment options exist that have the potential to alter the stochastic drift of a pollutant. The investments have capital and operating costs and are irreversible once constructed. The stochastically evolving pollutant induces stochastic damage. An option-pricing model provides a criterion for determining when it is optimal to adopt the investment with the highest option value. Option value, in this model, measures the expected present value in reduced damage, relative to doing nothing. If the investments are mutually exclusive, it is possible to obtain closed-form solutions for the barriers which would trigger investment. If the investments can be sequentially adopted, a methodology is developed to calculate option values for all possible combinations of adoption dates. To illustrate the option-pricing approach, a stylized analysis of investments to protect New York City's water supply is presented. Watershed management dominates filtration and, in the case of mutually exclusive investments, is initiated when the concentration of phosphorus reaches 22.80 μg/L.

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I. Introduction

Investments to improve water quality may require a commitment of highly specialized plant and equipment to a process whose future net benefits are uncertain. For example, a waste-water treatment plant may be designed based on a forecast of population growth and per capita water consumption. Depending on realized growth, water rates, and other factors, the plant may find itself at capacity sooner or later than anticipated. Consider a municipal government designing a system to provide safe drinking water to city residents. The best system may depend on the future quality of source water entering the system. Issues of timing, as well as scale and cost, may be critical to a project's value. Traditional cost-benefit analysis will not, in general, provide the appropriate method to evaluate investments which are costly to reverse and whose future net return is uncertain. Dixit and Pindyck (1994) and Trigeorgis (1996) suggest that option pricing models, originally developed to value stock options, may be equally important in the correct evaluation and timing of "real" investments.

The application of option pricing theory to real investments has been considered in a variety of contexts. Merton (1977) examined the relationship between financial options and real investment decisions by a firm. Myers (1977) showed that a firm's investment options determine, in part, its market value. Ownership of reserves of a nonrenewable resource are seen by Tourinho (1979) as options to produce that resource in the future. Arrow and Fisher (1974) viewed environmental preservation as providing a risk-neutral decision maker with a "quasi-option value."

There may be value in postponing an irreversible investment if additional information is expected in the future, as in Cukierman (1980) and Bernanke (1983), or in waiting until a stochastically evolving variable reaches a critical threshold, as in McDonald and Siegel (1986). In

McDonald and Siegel, the critical benefit-cost ratio could be considerably greater than one before it was optimal to exercise the option to invest.

Decisions to enter or exit an industry, when faced with fixed costs and a stochastically evolving price for output, have been examined by Brennan and Schwartz (1985) and Dixit (1989). Decisions on the optimal time to cut a stand of trees when price and the rate of growth in timber are stochastically evolving have been examined by Clarke and Reed (1989) and Reed and Clarke (1990). Conrad (1997) considers the optimal timing of an investment to slow global warming. Merton (1998) provides an overview on the pricing of options and the options inherent in real investments in his acceptance of the Nobel Prize in Economics.

The present paper extends this literature to a situation where two or more investment options exist which might alter the stochastic evolution of a pollutant. The pollutant is assumed to induce damage on a human population. The investments differ in terms of capital and operating costs as well as their ability to alter the mean drift and standard deviation rates of the pollutant. Our analysis provides a way to rank alternative, stochastic, water-quality investments.

The general model was motivated by an interest in the investment options facing New York City for the continued provision of safe drinking water to its residents. New York City is currently operating under a Memorandum of Agreement (MOA), allowing it to avoid filtration of water in its reservoirs west of the Hudson River, provided it is successful in implementing a watershed management plan that improves the quality of source water. By watershed management we will mean a set of actions or investments that might include the upgrading of municipal waste-water treatment plants, improving the management of animal waste and the use of fertilizers and pesticides on farms, the replacement of defective septic systems and the acquisition of fee simple title or conservation easements to land in order to form a "riparian buffer" around a lake or reservoir.

The rest of the paper is organized as follows. In the next section we present a model to evaluate two, mutually exclusive, investment projects that might alter the stochastic evolution of a pollutant. This model identifies critical barriers for each investment which in turn provide a criterion for ranking the investments. The third section considers the more realistic situation where investments might be adopted sequentially and where their values are interdependent. The fourth section applies both models to a stylized version of the problem facing New York City as it tries to maintain safe drinking water for its residents. The fifth section offers some conclusions and caveats when applying the option pricing model to investments to improve water quality.

II. Mutually Exclusive Investments

Let $C = C(t)$ denote the concentration of a pollutant at a particular location at instant t. Suppose that C has been stochastically increasing over time according to a process of Brownian motion as given by

$$dC = m\,dt + s\,dz \tag{1}$$

where $m > 0$ is the mean drift rate, $s > 0$ is the standard deviation rate and dz is the increment of a standard Wiener process, so that $dz = u(t)\sqrt{dt}$, where $u(t) \sim N(0,1)$. It should be noted that we are assuming a stochastic process where the concentration of the pollutant is expected to drift upward over time. On a short time scale stock pollutants may exhibit seasonality as a result of seasonal patterns in precipitation. Equation (1) would not be a good model of seasonal or cyclical stochastic behavior. When we go to the discrete-time version of Equation (1) in the next section, it might be argued that the average concentration during the period (perhaps across the relevant seasons) exhibits an upward drift over time. We will have some additional comments on alternative stochastic processes in Section V.

Suppose the pollutant causes damage when its concentration exceeds a threshold $\overline{C}$ and that instantaneous damage follows an exponential function according to

$$D = \begin{cases} \beta e^{\gamma(C-\overline{C})} & \text{if } C \geq \overline{C} \\ 0 & \text{if } C < \overline{C} \end{cases} \tag{2}$$

where β and γ are positive constants. At each instant damage increases at an increasing rate for $C > \overline{C}$.

Consider two investments which modify the drift in the concentration of the pollutant. Suppose Investment #1 is thought to modify the drift in the pollutant so that if it is adopted

$$dC = m_1\, dt + s_1\, dz \tag{3}$$

where $m > m_1$ and $s \geq s_1$. Water quality investments may have different capital and operating costs. Denote the capital costs of Investment #1 by
$K_1 > 0$ and its operating costs by $k_1 > 0$. If an investment is made (the project is implemented) we assume that it is irreversible and will be operated forever.

Assuming $C(t) > \overline{C}$, one can use It 's Lemma to show that damage is log normally distributed and with no treatment

$$dD = \alpha D\, dt + \sigma D\, dz \tag{4}$$

where $\alpha = \gamma m + (\gamma s)^2/2$ and $\sigma = \gamma s$. Equation (4) implies that damage evolves according to geometric Brownian motion, where α and σ are the mean drift and standard deviation rates, respectively. With treatment via Investment #1, damage would evolve according to

$$dD = \alpha_1 D\, dt + \sigma_1 D\, dz \tag{5}$$

where $\alpha_1 = \gamma m_1 + (\gamma s_1)^2/2$, $\sigma_1 = \gamma s_1$, and with $m > m_1$ and $s \geq s_1$, $\alpha > \alpha_1$ and σ ‡ σ_1.

If no investment is made, and the current observed damage is D(t), then the expected damage at $\tau \geq t$ is $D(\tau) = D(t)e^{\alpha(\tau - t)}$. The expected present value of damage from doing nothing is

$$\pi_0 = \int_t^\infty D(t)e^{\alpha(\tau-t)}e^{-\delta(\tau-t)}d\tau = \frac{D(t)}{(\delta-\alpha)} \tag{6}$$

where δ is the instantaneous discount rate and it is assumed that $\delta > \alpha$. We assume that δ is a risk-free rate reflecting the cost of borrowing by a large municipal government. If $\alpha \geq \delta$ the expected present value of damages is not defined, and it can be shown that it will be optimal to invest immediately.

If the capital and operating costs of Investment #1 are large and damages, though positive, are initially low, there will exist an interval of time, called the continuation region, where it is optimal not to invest. On the continuation region there is a value function $V_0 = V_0(D)$ which must satisfy the Hamilton-Jacobi-Bellman (H-J-B) equation that requires

$$\delta V_0 = D + \alpha D V_0' + (\sigma^2/2)D^2 V_0'' \tag{7}$$

This is a second-order, nonhomogeneous, ordinary differential equation (ODE) which has the solution

$$V_0(D) = \eta D^\varepsilon + D/(\delta - \alpha) \tag{8}$$

where $\varepsilon = (1/2 - \alpha/\sigma^2) + \sqrt{(\alpha/\sigma^2 - 1/2)^2 + 2\delta/\sigma^2}$ and with $\delta > \alpha$, $\varepsilon > 1$. [See Dixit and Pindyck (1994) for a discussion of this ODE and the fundamental quadratic.] We will show momentarily that the unknown constant, η, is negative. With this being the case, $V_0(D)$ has the following interpretation. The first term on the right-hand-side (RHS) of (8) is the *option value* of being able to optimally time the implementation of Investment #1 which will *reduce* the present value of expected damages. The second term on the RHS is the expected present value of doing nothing. With $\eta < 0$, option value, in this model, reduces the discounted expected damages *below* what they would be if there were no option to invest in water quality.

At some level of damage it will be optimal to incur capital costs K_1 and operating costs k_1 in exchange for switching from equation (4) to equation (5) where $\alpha > \alpha_1$. At the instant when Investment #1 is implemented, the expected present value becomes

$$V_1(D) = D/(\delta - \alpha_1) + K_1 + k_1/\delta \tag{9}$$

$V_1(D)$ is the sum of the present value of expected damages, when they are evolving according to equation (5), plus capital and discounted operating costs. At the instant when it is optimal to implement Investment #1 it must be the case that $V_0(D) = V_1(D)$ or

$$\eta D^{\varepsilon} + D/(\delta - \alpha) = D/(\delta - \alpha_1) + K_1 + k_1/\delta \tag{10}$$

This is called the *value-matching condition.*

When Investment #1 is adopted, the value function $V_0(D)$ must smoothly "hand off" to the value function $V_1(D)$. For a "seamless" transition, the derivatives of $V_0(D)$ and $V_1(D)$ must be identical at the critical level of damage that triggers the optimal implementation of Investment #1. This requires $V_0'(D) = V_1'(D)$, or

$$\varepsilon\eta D^{\varepsilon-1} + 1/(\delta-\alpha) = 1/(\delta-\alpha_1) \tag{11}$$

This is called the *smooth-pasting condition.* See Dixit (1993) for a discussion of the "art of smooth pasting."

The value-matching and smooth-pasting conditions are two equations which must hold at the critical damage level, D_1^*, which triggers Investment #1. These two equations permit us to solve for D_1^* and $\eta < 0$, the unknown constant defining option value. In this model we can get analytic solutions. Solving the smooth-pasting condition for η yields

$$\eta = \frac{(\alpha_1-\alpha)D^{1-\varepsilon}}{\varepsilon(\delta-\alpha)(\delta-\alpha_1)} \tag{12}$$

Note, with $\alpha > \alpha_1$, $\eta < 0$. Substituting the expression for η back into the value-matching condition and solving for D_1^* yields

$$D_1^* = \frac{\varepsilon(\delta-\alpha)(\delta-\alpha_1)(K_1 + k_1/\delta)}{(\varepsilon-1)(\alpha-\alpha_1)} \tag{13}$$

If there were only one treatment option, we could go home. Starting at some low, but positive, level of damage, D(0), we could tell our water quality minions to monitor damage, and when D(t) reaches D_1^*, build Investment Option #1. Figure 1. shows a realization D(t) reaching a hypothetical $D_1^* = 0.3$ at $t^* = 52$. For $t \geq 52$ damage evolves according to equation (3). In this hypothetical example $\alpha > 0 > \alpha_1$, and we see damages drifting downward from $D_1^* = 0.3$.

Often, there is more than one possible investment, and it would be useful to determine which investment dominates on an expected present-value basis, and when that investment should be implemented. For simplicity, suppose there is only one other investment, call it Investment #2. Suppose the capital and operating costs of Investment #2 are K_2 and k_2, respectively, and suppose it promises to change the drift in the pollutant to $dC = m_2\, dt + s_2\, dz$ and the drift in damages to

$$dD = \alpha_2 D\, dt + \sigma_2 D\, dz \tag{14}$$

where $\alpha_2 = \gamma m_2 + (\gamma s_2)^2/2$ and $\sigma_2 = \gamma s_2$. Given the analysis of Investment #1 we can simply state that the critical damage barrier for Investment #2 is

$$D_2^* = \frac{\varepsilon(\delta - \alpha)(\delta - \alpha_2)(K_2 + k_2/\delta)}{(\varepsilon - 1)(\alpha - \alpha_2)} \tag{15}$$

Given the expressions for D_1^* and D_2^*, and the role these barriers play in the timing of investment, we can say that Investment #1 dominates Investment #2 if $D_1^* < D_2^*$, and that Investment #2 dominates Investment #1 if $D_1^* > D_2^*$. Canceling the common terms we see that Investment #1 dominates if

$$\frac{(\delta - \alpha_1)(K_1 + k_1/\delta)}{(\alpha - \alpha_1)} < \frac{(\delta - \alpha_2)(K_2 + k_2/\delta)}{(\alpha - \alpha_2)} \tag{16}$$

and Investment #2 dominates if

$$\frac{(\delta - \alpha_1)(K_1 + k_1/\delta)}{(\alpha - \alpha_1)} > \frac{(\delta - \alpha_2)(K_2 + k_2/\delta)}{(\alpha - \alpha_2)} \tag{17}$$

We can see from (16) and (17) that the option pricing model has led, in this case, to a modified present value rule. Basically, $(K_i + k_i/\delta)$, i=1,2, is the standard present value calculation for the cost of the ith investment. These present values are modified by the term $(\delta - \alpha_i)/(\alpha - \alpha_i)$

which measures the relative decline in the mean drift rate of damage from the benchmark of no treatment.

We could expand the set of investments to more than two and provided that $\delta > \alpha > \alpha_i$ we could rank them through a pairwise comparison similar to that in (16) or (17). Option pricing theory has thus provided the theoretically correct way to rank investments which will alter the stochastic drift of a pollutant.

III. Interdependent Investments

The previous section assumed that Investment #1 and Investment #2 were mutually exclusive; you could implement one or the other, but not both. In reality it may be optimal to adopt one investment and then, depending on the evolution of the pollutant, C(t), implement the second investment at a later date.

The situation of multiple, interacting investments is discussed in Trigeorgis (1996, Chapter 7) where it is shown "that the value of an incremental option, in the presence of other options, is generally less than its value in isolation." In this section we will lay out a methodology for numerically estimating the value of two interacting investments that might be implemented at T_1 and T_2. While we will not obtain the elegant closed-form expressions for option value that emerged in the previous section, we will derive formulas for computing their expected present value. These formulas can be applied to the investment options of watershed management and filtration, as faced by New York City.

Consider the world of a water quality modeler whose has the ability to run any number of stochastic simulations. A particular simulation will be called a "realization," and suppose the modeler will run i=1,2,...,N realizations, where N is a large number. The concentration of the pollutant in the ith realization is assume to evolve according to

$$C_{i,t+1} - C_{i,t} = m + sU_{i,t} \quad (18)$$

where $U_{i,t}$ is a the standard normal variate for realization i in period t. The damage associated with concentration $C_{i,t}$ will again be given by

$$D_{i,t} = \begin{cases} \beta e^{\gamma(C_{i,t} - \bar{C})} & \text{if } C_{i,t} \geq \bar{C} \\ 0 & \text{if } C_{i,t} < \bar{C} \end{cases} \quad (19)$$

At some point in time it may be optimal to implement Investment #1. Denote that period as T_1. In the period when Investment #1 is adopted the concentration of the pollutant begins its evolution according to

$$C_{i,t+1} - C_{i,t} = m_1 + s_1U_{i,t} \quad (20)$$

While the concentration of the pollutant will evolve differently after the adoption of Investment #1, damages are still calculated according to equation (19).

Now suppose Investment #2 is adopted in T_2. The evolution of the pollution stock now follows

$$C_{i,t+1} - C_{i,t} = m_2 + s_2U_{i,t} \quad (21)$$

Suppose the horizon of interest is $T \geq T_2 > T_1 \geq 0$; that is, it is always optimal to adopt Investment #1 before Investment #2. (This does not have to be the case, but it will reduce the number of combinations (T_1,T_2) which must be stochastically evaluated.) With T, T_2, and T_1 given, our modeler could generate N realization, $C_{i,t}$, and calculate the associated damages, $D_{i,t}$, i=1,2,...,N and t=0,1,...,T. For a particular realization, the present value of damage could be calculated as

$$D_i = \sum_{t=0}^{T} \rho^t D_{i,t} \tag{22}$$

where $\rho = 1/(1 + \delta)$ and δ is the periodic rate of discount. A simple average yields an estimate of expected discounted damage if Investment #1 is implemented in T_1 and Investment #2 is implemented in T_2. Let

$$E\{D_{T_1,T_2}\} = (1/N)\sum_{i=1}^{N} D_i \tag{23}$$

denote the expected present value of damage from the evolving concentration of pollution when Investment #1 is adopted at T_1 and Investment #2 is adopted at T_2.

If Investment #1 is adopted at T_1, the present value of construction and operating costs are given by

$$COST_1 = \rho^{T_1}[K_1 + k_1 \sum_{t=T_1+1}^{t=T_2-1} \rho^t] = \rho^{T_1}[K_1 + k_1(\rho^{T_1} - \rho^{T_2-1})/\delta] \tag{24}$$

Formula (24) assumes that when Investment #2 is implemented, Investment #1 is abandoned with no scrap value and no further cost. The present value of costs for Investment #2, adopted at T_2, is

$$COST_2 = \rho^{T_2}[K_2 + k_2(\rho^{T_2} - \rho^{T-1})/\delta] \tag{25}$$

Formula (25) assumes that Investment #2 is implemented at T_2, operated until T-1, and abandoned with no scrap value or decommissioning cost.

Finally, let $E\{D_{T_1}\}$ denote the expected present value of damage when only Investment #1 is implemented in T_1, and $E\{D_{T_2}\}$ denote the expected present value of damage when only Investment #2 is implemented in T_2. Then, the option value of Investment #1, for given T_1, T_2, and T, may be calculated as

$$OV_1(T_1, T_2) = E\{D_{T_2}\} - E\{D_{T_1,T_2}\} - COST_1 \tag{26}$$

and the option value of Investment #2 may be calculated as

$$OV_2(T_1, T_2) = E\{D_{T_1}\} - E\{D_{T_1,T_2}\} - COST_2 \tag{27}$$

In the next section we will calculate option values and critical barriers when watershed management and filtration are mutually exclusive and when they are potentially sequential, and thus interdependent. The latter analysis is done by varying T_1 and T_2 to determine the largest option value.

IV. New York City's Water Supply

To illustrate the option pricing approach to evaluating investments in water quality we consider the case of New York City and its recent decision to make investments in watershed management as opposed to filtration. It should be mentioned at the outset that this example glosses over many details of the problem and the options facing New York City. In calibrating the model, many of the parameters are uncertain. So it is perhaps best to view this example of the option pricing approach as a numerical illustration, as opposed to an accurate case study of the New York City water-supply, investment options.

Back in 1835 the City of New York made a decision to develop a new source of drinking water from the Croton River, in present day Westchester County, east of the Hudson River, and about 40 miles north of Manhattan. The Croton River watershed drains about 369 square miles, and today provides the City with about 10 percent of its drinking water (Ashendorff *et al.* 1997). The other 90 percent is drawn from the Catskill and Delaware reservoir systems, west of the Hudson River, in watersheds covering approximately 1,600 square miles. Altogether the New York City system provides about 1.3 billion gallons of drinking water per day to approximately nine million people in its service area [National Research Council (1999)].

New York City's drinking-water supply is currently not filtered, although the Croton supply is scheduled to be filtered pursuant to a stipulation entered into with the New York State Department of Health. Like Boston, Portland, Oregon, San Francisco and several other cities, New York City relied on the natural purity of source water from its hinterlands. In the past, such water only required disinfection with chlorine to provide high-quality, safe drinking water.

In 1986, when Congress amended the Safe Drinking Water Act (SDWA), it called for regulations to address public health concerns over disinfection byproducts and microbial pathogens, such as *Giardia lamblia.* The Safe Water Treatment Rule, promulgated in June of 1989, required all public surface water supplies to be filtered unless the supplier can demonstrate it will "maintain a watershed control program which minimizes the potential for contamination by *Giardia* cysts and viruses in the source water." New York City does not view the filtration of the Catskill and Delaware systems as necessary. After months of negotiation, a Memorandum of Agreement (MOA) was signed on January 21st, 1997 by the City, communities in the Catskill/Delaware watershed, New York State, the EPA, and other environmental organizations. The MOA allows the City to avoid filtration of the Catskill/Delaware water supply provided it implements a broad set of watershed management investments and practices. The MOA runs through April, 2002. Continued avoidance of filtration will require the City to demonstrate that its watershed management program is effective in improving or maintaining the quality of water in the various reservoirs in the Catskill/Delaware system.

New York City has desperately sought to avoid filtration because of the high costs. It has estimated that filtration of the Catskill/Delaware systems would involve a capital cost of $6.0 billion and an operating cost of $350 million per year [Hevesi (1999)]. These cost estimates were disputed in the National Research Council (1999) report, where it was noted that an EPA-appointed panel concluded that the filtration of the Catskill/Delaware system might be achieved for as little as one-half of the City's estimate. Even at a capital cost of K_2 = $3.0 billion and an annual operating cost of k_2 = $175 million, filtration is more expensive than the City's estimate of K_1 = $500 million in capital cost and at least k_1 = $12 million in annual operating costs for the watershed management programs it has agreed to in the MOA [Hevesi (1999)]. In our illustration, watershed management becomes Investment #1, and filtration Investment #2.

In addition to *Giardia lamblia, Cryptosporidium parvum,* and various viruses and bacteria, surface water supplies need to limit the amount of bioavailable phosphorus. Bioavailable phosphorus will induce algae growth. High algal levels will increase total organic carbon (TOC). When source water that is high in TOC is treated with chlorine, four species of Trihalomethanes (THMs) will result. THMs are also called disinfection by-products (DBPs). In addition to causing taste, odor and color problems, certain DBPs have been identified as potential carcinogens in both toxicological and epidemiological studies [Black *et al.* (1996)].

While the various microbial agents (viruses, bacteria and protozoa) are of concern in the Catskill/Delaware system, phosphorus loadings and the resulting chain from algae, to TOC, to DBPs, seems to be the main focus of watershed management. As such, we adopt phosphorus as our state variable and, despite the potential for confusion, let $C = C(t)$ represent the concentration of phosphorus (in μg/L).

The concentration of phosphorus is especially critical during the growing season for algae (May - October). Limited annual data exist for the reservoirs in the Catskill/Delaware system prior to the commencement of watershed management practices. Based on data from the New York City Department of Environmental Protection (1997), for the period 1991 through 1996, the annual average rate of increase in phosphorus for all reservoirs in the Catskill/Delaware system was $m = 1.00$ (μg/L) with an annual standard deviation of $s = 2.25$ (μg/L).

The MOA has committed New York City to (1) improve municipal wastewater treatment plants in the Catskill/Delaware watersheds, (2) improve individual septic systems in those watersheds, (3) improve farm management practices in the use and handling of fertilizers and animal waste, and (4) purchase land or conservation easements that would create a system of riparian buffers around the reservoirs. At this point it is not clear whether these and other practices will reverse the trend in phosphorus concentration. Continued avoidance of filtration will hinge on their success. Lets suppose that collectively these investments and practices would result in $m_1 = -1.00$, but with the same standard deviation as in the period 1991 - 1996, that is, $s_1 = 2.25$.

Filtration, at least with regard to TOC, is something of silver bullet. While it will not solve problems with regard to microbial agents, filtration can remove TOC before chlorination and lessen the need for the management of phosphorus, at least with regard to the quality and safety of drinking water. We assume that filtration would drastically decrease TOC, and set $m_2 = -20.00$, and that it would do so with certainty, so that $s_2 = 0$.

Even more speculative than the mean drift and standard deviation rates are the parameters of the damage function. The current New York State guideline for phosphorus in source water is 20 μg/L. The National Research Council (1999) recommends that this be lowered to 15 μg/L. We adopt this recommendation and assume that for $C < \bar{C} = 15$ μg/L that damage is zero. By focusing on phosphorus, we are assuming that with
$C < \bar{C}$, chlorination and watershed management practices will be adequate to control microbial agents.

Suppose at $\bar{C} = 15$ μg/L that the damage from poor taste, odor, color, and the risk of cancer is \$40 million per year. This number is admittedly speculative, but with an additional cancer case evaluated at \$4.1 million by Magat *et al.* (1991), and given that the Catskill/Delaware system is providing approximately 90 percent of the drinking water to nine million people, $\beta =$ \$40 million per year at $\bar{C} = 15$ μg/L does not strike us as unreasonable. If the willingness-to-pay to improve water quality and reduce cancer risks when $C = 30$ μg/L is \$120 million, then this would imply, from the damage function, that $\gamma = \ln(120/40)/(30 - 15) \approx 0.07$.

The only remaining parameter for a base-case calibration is the discount rate, δ. The values for m, s, and γ imply that $\alpha \approx 0.08$ and for convergence of our present value calculations we adopt the somewhat high, but not unreasonable, annual discount rate of $\delta = 0.09$ (or nine percent per annum). When Investment #1 and Investment #2 are mutually exclusive, the base-case calibration and the resulting values for α, α_1, α_2, σ, σ_1, σ_2, ε, $D_1^*, D_2^*, C_1^*, C_2^*$, η_1, η_2, and option values are summarized in Table 1.

The first thing to note in Table 1 is that $D_1^* =$ \$69 million and is less than $D_2^* =$ \$513 million, so Investment #1 (watershed management) dominates Investment #2 (filtration). Perhaps more useful to managers would be the critical thresholds for phosphorus concentration.

Given the form of the damage function, and our closed-form solutions for D_1^* and D_2^*, we can calculate the critical phosphorus concentrations as

$$C_i^* = \frac{\ln\left[\frac{\varepsilon(\delta-\alpha)(\delta-\alpha_i)(K_i + k_i/\delta)}{(\varepsilon-1)(\alpha-\alpha_i)\beta}\right]}{\gamma} + \overline{C} \tag{28}$$

i=1,2. Note that if $D_1^* < D_2^*$, then $C_1^* < C_2^*$. In the Base Case $C_1^* = 22.80$ μg/L is a level which has been exceeded in the Cannonsville Reservoir in the Delaware Watershed and in six reservoirs in the Croton Watershed [New York City Department of Environmental Protection (1997)]. Reservoirs whose annual geometric mean concentration, plus one standard deviation, exceed 20 μg/L, for two consecutive five year intervals, are classified as "phosphorus restricted," and are targeted for efforts to reduce their phosphorus loading. Given the cost of filtration, the phosphorus concentration would have to reach $C_2^* = 51.47$ μg/L before filtration would be optimal.

The option values are listed as OV#1 and OV#2, for watershed management and filtration, respectively. These are calculated according to $OV\#i = -\eta_i D(t)^{\varepsilon}$, and assume that the current level of damage is D(t) = β = $40 million per year (at C = 15 μg/L). With $\eta_i < 0$, the option values are actually negative. (Remember, in this model, treatment options reduce the expected discounted damage of doing nothing.) Because of convention, we multiply by minus one, and list option value as positive. As expected, watershed management, the dominant investment, has the higher option value, and would be implemented first when C(t) reaches 22.80 μg/L, or equivalently, when D(t) reaches $69 million per year.

To get a feel for the sensitivity of dominance and the timing of Investments #1 and #2, Table 2 reports on the elasticities of $D_1^*, C_1^*, OV\#1, D_2^*, C_2^*$, and OV#2 to changes in selected parameters. In the row labeled "Parameter," we list the new value of the parameter which was changed. All other parameters were kept at their base-case values and the changes in

$D_1^*, C_1^*, OV\#1, D_2^*, C_2^*$, and OV#2 were measured from their base-case values. Elasticity is defined as the ratio of the percentage in a variable to the percentage change in a parameter. If the sign of an elasticity is negative, it means the variable increases (decreases) when the parameter decreases (increases). The size of the elasticity indicates the relative sensitivity of a variable to a parameter. If the absolute value of an elasticity is greater than one, it implies that the variable had a greater percentage change than the parameter.

Increases in m, s, or γ had the potential to cause α to exceed δ and result in the immediate adoption of the dominant investment. Changes in m_1, s_1, m_2, and s_2 resulted in very small changes in the critical barriers and option values. The parameters β and δ resulted in more significant changes, with option value particularly sensitive to changes in β and the near term, dominant Investment #1 relatively more sensitive to δ. Specifically, if β where to go up by one hundred percent, the value of both options would go up by 110 percent. If the discount rate were to increase by one hundred percent, Investment #1 would be likely to be adopted later, because the critical barrier D_1^* would increase by 184 percent.

If the capital or operating costs of an investment increase, it would increase the damage and concentration barriers and reduce option value. These elasticities, however, are all less than one in absolute value. If sensitive parameters are defined as those where a change might induce the immediate adoption of the dominant investment, or which have elasticities greater than one, then the model is sensitive to the values for m, s, β, γ, and δ and relatively insensitive to m_1, s_1, m_2, s_2, K_1, k_1, K_2, k_2, and $\overline{C}$.

When watershed management and filtration are sequential we calculated their option values for various combinations of T_1 and T_2 according to equations (26) and (27). For each combination of T_1 and T_2, N=1,000 realizations were run using a MATLAB (Version 5) program which is available from the first author. Using the base-case parameters in Table 1, we set an upper bound of T = 100 years and $C_{i,0} = 15$ µg/L for all realizations.

Of the combinations where $100 \geq T_2 > T_1 \geq 1$, the largest combined option value, $OV_1(T_1,T_2)+OV_2(T_1,T_2)$, occurred when $T_2=100$ and $T_1=2$. For these values, $OV_1(T_1,T_2)=\$2.8034$ billion and $OV_2(T_1,T_2)$ was negative, indicating that you would never build the filtration plant. Because an option does not have to be exercised, the option value of filtration is actually the maximum of $OV_2(T_1,T_2)$ or zero; in this case zero. Thus, the maximum combined option value was \$2.8034 billion.

Of the combinations where $100 \geq T_1 > T_2 \geq 1$, the largest combined option value occurred when $T_1=100$ and $T_2=30$. For these values, $OV_1(T_1,T_2)=\$28{,}902{,}000$ and $OV_2(T_1,T_2)=1.9658$ billion. The maximum combined value was \$1.9947 billion. This value was dominated by the previous maximum at $T_2=100$ and $T_1=2$, and the sequential investment with the highest expected present value involves watershed management at $T_1=2$ and no filtration.

The maximum sequential amount of \$2.8034 billion, while large, is less than the base-case value of \$4.43 billion attributed to watershed management when the investments were mutually exclusive. This is consistent with the previous quote from Trigeorgis (1996).

V. Conclusions and Caveats

This paper has attempted to show the relevance of option pricing in the evaluation of investments to improve water quality. Often an investment in water quality holds the promise of reducing the concentration of a pollutant, but only in a stochastic sense, where the expected drift in concentration would be less than what it would have been had nothing been done. If water quality investments require a commitment of specialized capital, then option pricing would provide the correct way to rank investments and determine the barrier which would trigger the adoption of the dominant investment. To illustrate this approach we attempted to rank and determine the barriers which would trigger watershed management or filtration of the Delaware and Catskill reservoirs which provide New York City with about 90 percent of its drinking water. Two perspectives were taken. If the investments were mutually exclusive it was possible to derive analytical expressions for option value and the critical barriers (concentration or damage) which would trigger the implementation of a particular project. If the projects were both feasible and could be adopted sequentially, they are said to be interdependent. Interdependent projects will normally have a lower option value than when they are evaluated individually and regarded to be mutually exclusive. Perhaps the most interesting result from our stylized analysis of the investments to maintain safe drinking water for New York City, was that the option value of filtration fell to zero if both watershed management and filtration were sequentially feasible. These numerical results were dependent upon correct assessments for the costs of filtration and, most importantly, on the assessment of the health costs of water with elevated levels of disinfection by-products. If cancer risks or the cost of "an additional statistical cancer case" are higher, filtration might be a more favorable option.

While the option pricing approach has intuitive appeal, it has several shortcoming as well. First, there will be relatively few instances where one can obtain explicit (or closed-form) expressions for the critical barrier. Indeed, the Brownian motion of the pollutant's concentration was deliberately matched with our exponential damage function because we knew, in advance, that it would result in damages evolving according to geometric Brownian motion and would allow for closed-form expressions for D_1^* and D_2^*. While there are many other stochastic processes which could be used to describe the evolution of a pollutant or its damage, one will typically have to settle for numerical analysis when determining the values or barriers that would trigger an investment. For example, Ethier (1999) examines the value of generating capacity when the price of electricity evolves according a mean-reverting process with an occasional, Poisson-generated, jump. There is no analytic solution for the value function, but it may be approximated by an exponential series.

Second, to statistically distinguish between competing stochastic processes, or to precisely estimate the parameters of a particular process, one would like a long time series of consistent measures on the state variable (concentration or damage). Such data, on an annual basis, are seldom available, making it difficult to determine, with a high degree of precision, the likely drift of a pollutant if no investment is undertaken. The mean and standard deviation after making an investment is likely to be even more speculative if a technology and practice are new or untried at the proposed scale of treatment.

Third, option pricing models, applied to real investments, are more tractable when dealing with an infinite horizon. We assumed that if an investment was made, it would be operated forever. In reality, the operating life of plant and equipment may be 20 to 30 years and its performance (effectiveness) may decline with age. Finite-horizon models with time-varying parameters can be formulated, but as with the more exotic stochastic processes, they are not likely to lead to closed-form solutions with clean economic interpretations as to the ranking and timing of alternative investments. When investments are interdependent, numerical analysis, such as that implied in Section III, will typically be necessary.

Despite these shortcomings, we feel that the option-pricing approach provides the theoretically correct way to rank projects that might stochastically improve water quality. It provides a logical framework for integrating stochastic models of water quality with economic measures of cost and damage. To be done well, it will require careful monitoring of the current drift in water quality and the design and monitoring of pilot projects which would provide estimates of mean drift and variance rates for investment projects built to scale. Economists need to be able to estimate not only the capital and operating costs of alternative options, but the damage from color, taste, and the health risks associated with different quality drinking water. This is not a trivial nor easy research agenda, but given the potential costs of filtration facing Boston, San Francisco, Portland, Oregon, as well as New York City, it may be a wise investment.

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Figure 1. A Realization, D(t), and the Critical Barrier, $D_l^* = 0.3$

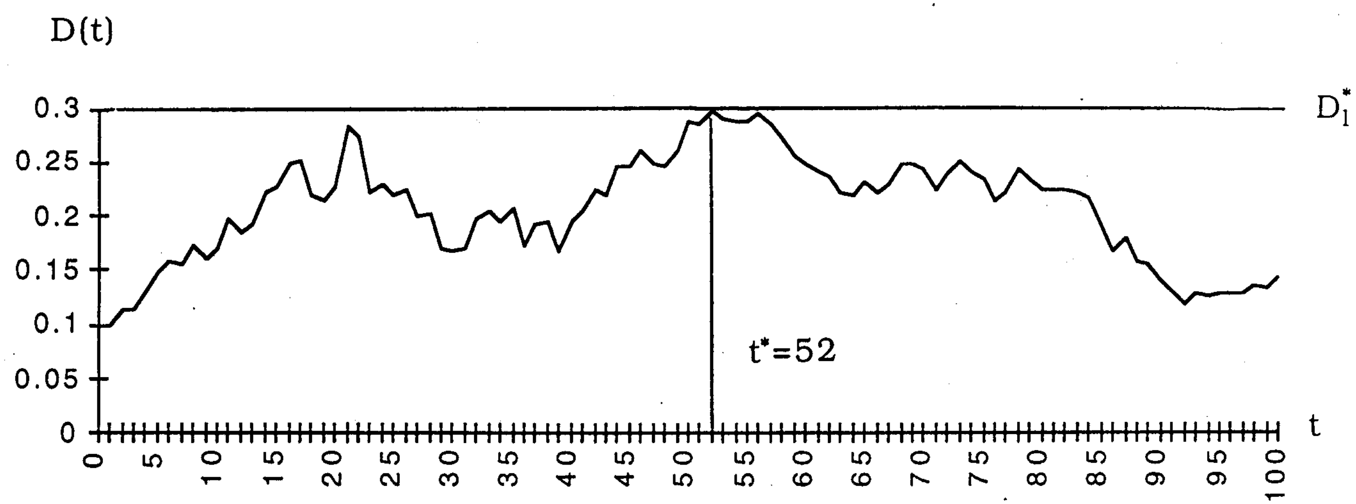

Table 1. Base-Case Results

Parameters:	Calculated Values:
$m=1$	$\alpha=0.08$
$s=2.25$	$\alpha_1=-0.06$
$m_1=-1$	$\alpha_2=-1.40$
$s_1=2.25$	$\sigma=\sigma_1=0.16$
$m_2=-20$	$\sigma_2=0$
$s_2=0$	$\varepsilon=1.08$
$\beta=\$40,000,000$	$D_1^*=\$69,000,000$
$\gamma=0.07$	$D_2^*=\$513,000,000$
$\delta=0.09$	$C_1^*=22.80\ \mu g/L$
$K_1=\$500,000,000$	$C_2^*=51.47\ \mu g/L$
$k_1=\$12,000,000$	$\eta_1=-27.64$
$K_2=\$3,000,000,000$	$\eta_2=-24.73$
$k_2=\$175,000,000$	OV#1=\$4,430,000,000
$\overline{C}=15$	OV#2=\$3,960,000,000

Comment: Option values (OV#1 and OV#2) are calculated according to OV#i $= -\eta_i D(t)^{\varepsilon}$, for i=1,2 and $D(t)=\beta=\$40,000,000$.

Table 2. The Elasticities of D_1^*, C_1^*, OV#1, D_2^*, C_2^*, and OV#2 with respect to selected parameters.

Parameter	m_1=-2	s_1=4.5	m_2=-40	β=8E7	δ=0.18	K_1=1E9	k_1=2.4E7	K_2=6E9	k_2=3.5E8
D_1^*	-0.02	+0.02	0	0	+1.84	+0.79	+0.21	0	0
C_1^*	-0.01	+0.02	0	-0.43	+0.65	+0.36	+0.12	0	0
OV#1	+0.02	-0.02	0	+1.11	-0.34	-0.05	-0.02	0	0
D_2^*	0	0	-0.002	0	+0.68	0	0	+0.61	+0.39
C_2^*	0	0	-0.007	-0.19	+0.14	0	0	+0.13	+0.09
OV#2	0	0	+0.0025	+1.11	-0.70	0	0	-0.04	-0.03

Comments:

(a) Elasticity=(Percentage Change in Value)/(Percentage Change in Parameter). Change is measured from the Base-Case Value and Parameter.

(b) Increases in m, s, or γ can cause $\alpha > \delta$, implying it is optimal to adopt Option #1 immediately.

(c) Increasing s_2 from zero to one caused an insignificant increase in D_2^* and C_2^* and an insignificant decrease in OV#2.

(d) If sensitive parameters are defined as those which, if changed by 100%, can induce immediate adoption of Option #1 or which generate elasticities greater than one, then m, s, β, γ, and δ are sensitive paramters and need to be estimated as precisely as possible. By this definition, calculated values are relatively insensitive to changes in m_1, s_1, m_2, s_2, K_1, k_1, K_2, k_2, and $\overline{C}$.